

To Derive the Mathematical Relations for the Models Taken for the Study

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Abstract: *The mathematical modeling of solar cells is critical for analyzing and optimizing their performance under various operating conditions. This work focuses on the two prevalent models used to represent the electrical behavior of photovoltaic devices: the single diode and double diode models. Both models are based on an equivalent electrical circuit that includes a photocurrent source, diodes representing recombination losses, and resistive elements representing internal losses. The single diode model simplifies recombination losses into one diode, offering computational efficiency, while the double diode model introduces a second diode to capture additional recombination mechanisms for greater accuracy. This research compares these models by deriving their mathematical equations, discussing their parameters, and evaluating their respective merits. The insights gained assist in selecting an appropriate model for system simulation, design, and performance prediction, depending on the complexity and accuracy requirements in photovoltaic applications.[1].*

Keywords: Mathematical Model, Diode, photovoltaic cells

I. INTRODUCTION

The mathematical modeling of solar cells is fundamental to understanding and predicting their electrical behavior under different environmental conditions such as irradiance and temperature. These models typically represent the solar cell as an equivalent electrical circuit, capturing key physical phenomena like photocurrent generation, diode junction behavior, and resistive losses. The most common approaches are the single diode and double diode models. The single diode model uses one diode to represent their combination losses in the p-n junction, making it simpler and widely used for general analysis. In contrast, the double diode model incorporates two diodes to accurately model different recombination mechanisms within the cell, providing better precision especially under non-ideal conditions. Mathematical relationships derived from these models form the basis for simulation, parameter extraction, and performance optimization in photovoltaic system design. Such models enable the translation of solar irradiance and temperature inputs into current-voltage (I-V) characteristics essential for evaluating cell efficiency and power output.[2][3].

A. Mathematical relation of a single diode solar cell

The mathematical relation of a single diode solar cell can be derived step by step starting from the equivalent circuit, which includes a current source representing the photo generated current, a diode, a series resistance, and a shunt resistance.

Step-by-Step Derivation of the Single Diode Solar Cell Equation

Equivalent Circuit and Components:

I_L : Light-generated current (photocurrent), proportional to solar irradiance.

I_0 : Diode reverse saturation current (dark current).

n : Ideality factor of the diode (typically between 1 and 2)

R_s : Series resistance representing internal resistive losses.

R_{sh} : Shunt resistance representing leakage current paths.

V : Voltage across the solar cell terminals.

I : Output current of the solar cell.

Current Components Using Kirchhoff's Law:

The total current flowing out of the solar cell is given by:

$$I = I_L - I_D - I_{sh}$$

where I_D is the current through the resistance and I_{sh} is the current through the shunt resistance

Diode Current Using Shockley Equation:

The diode current follows the ideal diode equation:

$$I_D = I_0 \left[\exp\left(\frac{V_j}{nV_T}\right) - 1 \right]$$

where

I_0 is the reverse saturation current,

V_j is the junction voltage across the diode,

n is the ideality factor,

$$V_T = \frac{kT}{q}$$

is the thermal voltage (Boltzmann constant \times temperature / electron charge).

Voltage across the Diode and Series Resistance:

The terminal voltage is related to the junction voltage V_j by the series resistance:

$$V_j = V + IR_s$$

Shunt Current Through R_{sh}

The current through the shunt resistor is:

$$I_{sh} = \frac{V_j}{R_{sh}} = \frac{V + IR_s}{R_{sh}}$$

Complete Current-Voltage Relation:

Substituting I_D and I_{sh} into the Kirchhoff equation:

$$I = I_L - I_0 \left[\exp\left(\frac{V + IR_s}{nV_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}$$

This implicit transcendental equation expresses the single diode solar cell current I as a function of the terminal voltage, incorporating physical solar cell parameters and resistive losses. The five parameters I_L , I_0 , n , R and R_{sh} describe the electrical behavior of the solar cell under illumination and can be extracted from the I-V curve experimentally or through fitting.

Summary of Derived Equation:

$$I = I_L - I_0 \left[\exp\left(\frac{V + IR_s}{nV_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}$$

where

I_s is the output current,

V is the output terminal voltage,

I_L is the light generated current,

I_0 is the diode saturation current,

n is the diode ideality factor,

R_s is the series resistance,

R_{sh} is the shunt resistance,

$$V_T = \frac{kT}{q}$$

is the thermal voltage.

This formula forms the basis for the single diode model in photovoltaic analysis and simulation.

If needed, numerical methods like the Newton-Raphson algorithm are often used to solve this implicit equation for given V . [4][5][6][7]

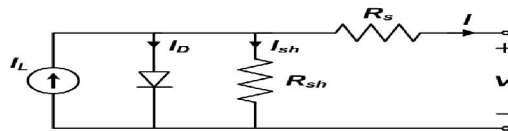


Fig.1. Circuit Diagram of Single Diode Solar Cell

B. Mathematical relation of a double diode solar cell

The mathematical relation of a double diode solar cell is derived by extending the single diode model to include two diodes that represent different recombination mechanisms in the solar cell. This improves the accuracy of the model, especially at low voltages.

Step by Step Derivation of the Double Diode Solar Cell Equation

Equivalent Circuit and Components:

The double diode solar cell model consists of:

I_L : Light-generated current (photocurrent).

I_{D1} : Current through the first diode representing diffusion current.

I_{D2} : Current through the second diode representing recombination current.

R_s : Series resistance.

R_{sh} : Shunt resistance.

V : Terminal voltage.

I : Output current.

Current Components and Kirchhoff's Law:

The total current is:

$$I = I_L - I_{D1} - I_{D2} - I_{sh}$$

Where I_{sh} is the shunt current.

Diode Currents Using Shockley Equation:

The currents through the diodes are:

$$I_{D1} = I_{01} \left[\exp \left(\frac{V_j}{n_1 V_T} \right) - 1 \right]$$

$$I_{D2} = I_{02} \left[\exp \left(\frac{V_j}{n_2 V_T} \right) - 1 \right]$$

where

I_{01}, I_{02} are the reverse saturation currents,

n_1, n_2 are the ideality factors of diodes 1 and 2,

$V_T = \frac{kT}{q}$ is the thermal voltage,

$V_j = V + IR_s$ is the voltage across the diodes and series resistance.

Shunt Current Through

Complete Current-Voltage Relation

Combining all terms, the current is:

$$I = I_L - I_{01} \left[\exp\left(\frac{V + IR_s}{n_1 V_T}\right) - 1 \right] - I_{02} \left[\exp\left(\frac{V + IR_s}{n_2 V_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}$$

This expression implicitly relates the terminal current and voltage of the double diode solar cell accounting for two diode effects and resistive losses.

Summary of Double Diode Equation:

$$I = I_L - I_{01} \left[\exp\left(\frac{V + IR_s}{n_1 V_T}\right) - 1 \right] - I_{02} \left[\exp\left(\frac{V + IR_s}{n_2 V_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}$$

Where the parameters are as defined above.

This double diode model better captures recombination losses in the depletion region and is especially useful in modeling real solar cells with higher accuracy compared to the single diode model. Due to the implicit nature of the equation, numerical methods like the Lambert W function or iterative solutions are typically used to solve for I given V . [8][9][10].

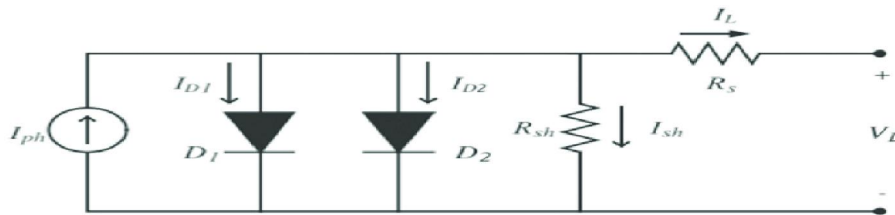


Fig.2. Circuit Diagram of Double Diode

Compare Mathematical Model of a single diode and double diode solar cells

The mathematical models of single diode and double diode solar cells have fundamental differences in their equations and parameters, reflecting the physical processes they represent:

Aspect	SingleDiodeModel	DoubleDiodeModel
Circuit Elements	Photocurrent source, one diode, series resistance R_s , and shunt resistance R_{sh} .	Photocurrent source, two diodes (each with own saturation current and ideality factor), series resistance R_s , shunt resistance R_{sh} .
Equation	$I = I_{ph} - I_0 \left(e^{\frac{q(V+IR_s)}{n_1 kT}} - 1 \right) - \frac{V+IR_s}{R_{sh}}$	$I = I_{ph} - I_{01} \left(e^{\frac{q(V+IR_s)}{n_1 kT}} - 1 \right) - I_{02} \left(e^{\frac{q(V+IR_s)}{n_2 kT}} - 1 \right) - \frac{V+IR_s}{R_{sh}}$

Number of Parameters	5 parameters: $I_{ph}, I_0, n,$	Typically 7 parameters: $I_{ph}, I_{01}, n_1, I_{02}, n$
Physical Representation	Models recombination losses with a typically single diode ideality factor.	Captures two recombination mechanisms separately, with different ideality factors.
Model Complexity	Simpler, easier to implement for real-time simulation.	More complex, requires additional parameter estimation but offers higher fidelity.
Accuracy	Good approximation under normal conditions but less accurate at low irradiance or non-ideal conditions.	More accurate over a wider range of operating conditions due to better recombination loss representation.

Derive the mathematical relation of a solar water heater

The mathematical relation of a solar water heater is derived based on energy balance principles involving the solar collector, the fluid in the system, and heat losses.

Step by Step Derivation of Solar Water Heater Mathematical Model

Energy Balance on the Solar Collector:

The useful thermal energy gained by the fluid in the collector, Q_u , can be expressed as:

$$Q_u = A_c F_r [G_t \tau \alpha - U_L (T_p - T_a)]$$

where

A_c is the collector area,

F_r is the collector heat removal factor,

G_t is the solar radiation on the collector surface,

τ and α are the transmittance and absorptance of the collector,

U_L is the overall heat loss coefficient,

T_p is the fluid temperature at the collector outlet or plate temperature,

T_a is the ambient temperature.

Fluid Temperature Differential Equation:

The change of temperature of the fluid flowing through the collector is given by:

$$m c_p \frac{dT_f}{dt} = Q_u - m c_p (T_f - T_i)$$

Where

m is the mass flow rate of the fluid,

c_p is the specific heat capacity of the fluid,

T_f is the fluid temperature,

T_i is the inlet fluid temperature,

t is time.

Storage Tank Energy Balance

The energy balance on the tank considers the inlet and outlet fluid temperatures, usage losses, and solar input:

$$m c_p (T_{out} - T_{in}) = Q_u - Q_{loss} - Q_{use}$$

where

Q_{loss} represents heat losses from the tank to the environment,

Q_{use} is the energy drawn for hot water usage.

Heat Loss Model:

Heat loss Q_{loss} can be modeled as:

$$Q_{loss} = U_t A_t (T_t - T_a)$$

where

U_t is the overall heat loss coefficient for the tank,

A_t is the tank surface area,

T_t is the tank temperature.

Over all System Equation:

Combining these relations and considering transient conditions yields differential equations that can be solved for temperature evolution and energy delivered by the solar water heater system.

This set of equations describes the performance of a solar water heater system based on collector efficiency, fluid properties, and environmental conditions. Numerical methods are generally used for transient simulation and optimization.

This model is essential for designing and predicting thermal output and efficiency of solar water heating systems.[15][16][17][18]

Derivation of Energy Balance for Flat Plate Collector Outlet Temperature Step by step

Energy Input and Output:

The energy absorbed by the collector is from solar radiation, and it is used to heat the fluid while the collector loses heat to the surroundings:

$$Q_u = Q_{in} - Q_{loss}$$

where

Q_u is the useful heat gained by the fluid,

$Q_{in} = A_c G_{\tau\alpha}$ is the absorbed solar radiation,

$Q_{loss} = A_c U_L (T_p - T_a)$ is the total heatloss,

A_c is the collector area,

G is the solar irradiance on the collector,

τ is the transmittance of the cover,

α is the absorptance of the absorber plate, U_L is the overall heat loss coefficient,

T_p is the plate temperature,

T_a is the ambient temperature.

Heat Transfer to the Fluid:

The useful energy gained by the fluid is related to the fluid mass flow and temperature rise:

$$Q_u = \dot{m} c_p (T_o - T_i)$$

where

\dot{m} is the mass flow rate of the fluid,

c_p is the specific heat capacity of the fluid,

T_i and T_o are the inlet and outlet fluid temperatures.

Heat Removal Factor and Collector Efficiency:

To account for in efficiencies in heat transfer between the plate and fluid, a heat removal factor F_r is introduced:

$$Q_u = A_c F_r [G\tau\alpha - U_L(T_i - T_a)]$$

Deriving Outlet Temperature:

Substitute Q_u in the heat transfer equation:

$$\dot{m}c_p(T_o - T_i) = A_c F_r [G\tau\alpha - U_L(T_i - T_a)]$$

Solving for outlet temperature T_o :

$$T_o = T_i + \frac{A_c F_r}{\dot{m}c_p} [G\tau\alpha - U_L(T_i - T_a)]$$

This equation explicitly relates the outlet temperature T_o of the fluid exiting the solar collector to the inlet temperature, solar radiation, collector properties, fluid flow, and ambient conditions.

The model balances absorbed solar energy, losses, and fluid heat gain, providing a standard formula widely used for flat plate solar collector performance evaluation and simulation. [19][20][21][22].

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